Superconductor to insulator transition: a short overview on recent ideas

C. Castellani

Collaborations
L. Benfatto and J. Lorenzana (Roma),
G. Seibold (Cottbus)
G. Lemarié (Toulouse), D. Bucheli (PhD, Roma)

References
Final fate of dirty superconductors by increasing disorder

Two energy scales

- pairing energy $\Delta_{\text{gap}}$: gap in tunneling (DOS) and optics
- superfluid stiffness $D_s$: phase coherence and superfluid response
Disappearing of SC by increasing disorder

• “Fermionic” vs “bosonic” mechanism

“Fermionic” mechanism (Finkelstein): disorder enhances Coulomb repulsion, pairing strength decreases, both $T_c$ and $\Delta$ go to zero $\Rightarrow$ FM or FI

“Bosonic” mechanism (Fisher, Ma & Lee, etc.) direct localization of Cooper pairs, finite pairing in the insulating phase
Thickness tuned SIT: amorphous Bi films

- $\Delta_{\text{gap}} \rightarrow 0$ as $T \rightarrow T_c$
- Insulator: Weakly localized electrons
- Small positive MR
- $\Delta_{\text{gap}} \neq 0$ at $T > T_c$
- Localized Cooper Pairs with activated transport $R = R_0 e^{T_0/T} \ T_0 \rightarrow 0$ at SIT
- Giant positive MR

Bosonic mechanism at work

Jim Valles, Leiden 2011
Thickness tuned SIT: amorphous Bi films

- $\Delta_{gap} \Rightarrow 0$ as $T \Rightarrow T_c$
- Insulator: Weakly localized electrons
- Small positive MR
- $\Delta_{gap} \neq 0$ at $T > T_c$
- Localized Cooper Pairs with activated transport $R = R_0 e^{T_0/T}$ where $T_0 \Rightarrow 0$ at SIT
- Giant positive MR

Bosonic mechanism at work $\neq$ from granular
Thickness tuned SIT: amorphous Bi films

- $\Delta_{\text{gap}} \Rightarrow 0$ as $T \Rightarrow T_c$
- Insulator: Weakly localized electrons
- Small positive MR
- $\Delta_{\text{gap}} \neq 0$ at $T > T_c$
- Localized Cooper Pairs with activated transport
- $R = R_0 e^{T_0/T}$, $T_0 \Rightarrow 0$ at $T_c$
- Giant positive MR

Trivial consequence of artificial nanostructure
Superconductor to insulator transition: bosonic mechanism at work

- Homogeneous InO, TiN, NbN films share the same features: $\Delta_{\text{gap}}$ stays $\neq 0$ above $T_c$, activated transport $R = R_0 e^{T_0/T}$, giant positive MR
- Are they inhomogeneous? Effectively inhomogeneous?
- General mechanism for inhomogeneity?
Ioffe and Mezard proposal for disordered superconductors

- low temperature glassy phase (with one-step replica symmetry breaking)

- self-generated inhomogeneity on a mesoscopic scale (much) larger than the scale of disorder and of pairing ($\xi_{sc}$)

Ioffe and Mezard, PRL 105, 037001 (2010);
Feigelman, Ioffe and Mezard, PRB 82, 18534 (2010)
Ioffe and Mezard proposal for disordered superconductors

• Ising model in a transverse random field (strong coupling pairing with \(<\sigma^x_i> = \) superconducting order parameter) on a Cayley tree with K-branching

\[ H = -\sum_i \xi_i \sigma_i^z - g/K \sum_{<ij>} \sigma_i^x \sigma_j^x \]

• Mean field- cavity method: recursion formula for the Weiss field acting on \(\sigma_i^x\):

\[ B_i = g/K \sum_j <\sigma_j^x> \equiv g/K \sum_j \frac{B_j}{\sqrt{\xi_j^2 + B_j^2}} \tanh \beta \sqrt{\xi_j^2 + B_j^2} \]

at low T only few paths contribute to pairing susceptibility from boundary to root \(\chi_{0L} = \partial<\sigma_0^x> / \partial h_L^x\) (by mapping to directed polymer problem)

\[
\begin{align*}
\text{boundary} \\
\text{root}
\end{align*}
\]
Ioffe and Mezard proposal for disordered superconductors

- Ising model in a transverse random field (strong coupling pairing with $\langle \sigma^x_i \rangle = $ superconducting order parameter) on a Cayley tree with K-branching

$$ H = - \sum_i \xi_i \sigma^z_i - g/K \sum_{<ij>} \sigma^x_i \sigma^x_j $$

- Mean field- cavity method: recursion formula for the Weiss field acting on $\sigma^x_i$

$$ B_i = g/K \sum_j <\sigma^x_j> \equiv g/K \sum_j \frac{B_j}{\sqrt{\xi^2_j + B^2_j}} \tanh \beta \sqrt{\xi^2_j + B^2_j} $$

Superconductivity on filaments: self-generated inhomogeneity on a mesoscopic scale
Ioffe and Mezard proposal for disordered superconductors

- Ising model in a transverse random field (strong coupling pairing with $<\sigma^x_i> = $ superconducting order parameter) on a Cayley tree with K-branching

\[ H = - \sum_i \xi_i \sigma_i^z - g / K \sum_{<ij>} \sigma_i^x \sigma_j^x \]

- Mean field- cavity method: recursion formula for the Weiss field acting on $\sigma^x_i$

\[ B_i = g / K \sum_j <\sigma^x_j> \equiv g / K \sum_j B_j \frac{B_j}{\sqrt{\xi_j^2 + B_j^2}} \tanh \beta \sqrt{\xi_j^2 + B_j^2} \]

Anomalous distribution of the local order parameter $s = <\sigma^x_i> : P(s) \propto s^{-\alpha}$ and non self-averaging properties
Ioffe and Mezard proposal for disordered superconductors

Various questions:

• Quasi-1d, non self-averaging and anomalous distribution will survive from Cayley tree to real lattices (2d)?
  (The replica symmetry breaking issue in finite d)

• Intrinsically strong coupling (bosonic) model: what for a fermionic model of superconductivity from weak to strong coupling? ⇒ attractive Hubbard model

• Price to pay: from cavity methods to mean field, however MF can be a reasonable description of the ordered phase
The attractive Hubbard model

• we consider the attractive Hubbard model with on site disorder

\[ H = -t \sum_{<ij>,\sigma} a^+_i a_j - |U| \sum_{i} n_{i\uparrow} n_{i\downarrow} - \sum_{i\sigma} \xi_i n_{i\sigma} \]

• and solve the mean field Bogoliubov-de Gennes eqs on a 2d finite cluster at T=0 with site dependent SC order parameter \( \Delta_i = |U| <a^+_i a_i^-> \), and \(-V_0<\xi_i<V_0\)

• We can study weak coupling with sizeable non locality \( \xi_{sc} \sim 10 \) lattice spacing and more parameters (e.g. filling)
The attractive Hubbard model

- Parameters: $t=1$, $U=1.5\div10$, $V_0=0.1\div3$, $<n>=0.1\,\div1$
- Various known results from previous BdG and Monte Carlo. Huge literature. See: Ghosal, Randeria and Trivedi, PRL(1998) and PRB (2001)
- Even for not too large $U$ (with $V_0\sim1$) superconductivity is established by coherence of local pairs. Cfr: Feigelman et al Ann.Phys (2010)
- Strong variations of local SC order parameter $\Delta_i$ ($\leq\Delta_{\text{gap}}$, spectral gap from local Density of States) “superconducting islands” (inhomogeneity)
The attractive Hubbard model

- Distribution of the local order parameter
  \[ s_i = 2\Delta_i/U \equiv 2\langle a^+_i a^+_i \rangle \leftrightarrow \langle \sigma^x_i \rangle. \]
  Note: \( \Delta_i \) is not the DOS gap, it is a measure of coherence.

- Predictions of IM in the ordered phase (\( g \geq \) critical value):
  \[ P(s) \sim s^{m_{typ}}/s^{1+m} \]
  for large \( s \), with \( m = m(K, T/V_0) < 1 \).
  “Unbounded” distribution but for the physical constraint \( s \leq 1 \).
  Averaged \( \langle s \rangle \gg s_{typ} \)
  with \( s_{typ} = \exp<\ln s> \)

- We find a qualitative agreement (broad \( P(s) \)), but a different distribution
Distribution of the order parameter

Very crowded:

Hubbard
\( U=5, g=t^2/UV_0=0.08, n=0.85 \)
\( U=5, g=0.2, n=1 \)

XX-Z
2DCMF (Monthus and Garel 2012), \( g=0.4 \)
MF, \( g=0.2 \) (dashed-dotted)
Cayley, \( K=3, g=0.2 \) (dotted)
Distribution of the order parameter

Universal by rescaling $R = (\ln s - \ln s_{typ}) / \sigma_s$

$\ln s_{typ} = \langle \ln s \rangle$

$\sigma_s^2 = \langle (\ln s - \ln s_{typ})^2 \rangle$

**Hubbard**
- $U=5, g=t^2/UV_0=0.08$, $n=0.85$
- $U=5, g=0.2, n=1$

**XX-Z**
- 2DCMF (Monthus and Garel 2012), $g=0.4$
- MF, $g=0.2$ (dashed-dotted)
- Cayley, $K=3, g=0.2$ (dotted)
Distribution of the order parameter

• Universal distribution:

• within our MF approach the rescaling seems to work independently of parameters in a wide range, with the variance $\sigma_s \sim -\ln s_{\text{typ}}$ increasing for $g \to 0$

• In the disordered phase analogy with directed polymer problem physics in 2D: $\ln s_L = -c L + L^\omega u$ with $\omega \approx 1/3 > 0$ ($\omega = 0$ on the Cayley tree)

$L =$ distance from ordered boundary, $u =$ random variable with Tracy-Widom distribution
magenta dashed dotted line, BdG with $|U| = 1.5$, $\langle n \rangle = 0.875$, $L = 25$, $g = 0.2$, i.e., $V_0 = 3.33$. 
Distribution of the order parameter

• Comparison with experiments

• Hypothesis: $\Delta(r)$ correlates with relative height of coherent peaks in local density of states in STM (Sacépé et al, Nature Phys. 7, 239, (2011): STM in InO films)

• Here we compare with data on NbN films from Tata Institute group (Pratap Raychaudhuri and collaborators)

Define peak height $h = (G_{\text{peak}} - G_{\text{min}})/G_{\text{min}} \propto \text{SC OP}$
Lemarié et al. PRB 87, 184509 (2013)
STM in NbN films
Lemarie et al. PRB 87, 184509 (2013)
STM in NbN films

Distribution of the local peak height $h \propto$ SC order parameter $s$
Lemarie et al. PRB 87, 184509 (2013)
STM in NbN films

Rescaled distribution of the local peak height $h \propto s = \text{SC OP}$

$$R_s = \frac{\ln s - \ln s_{\text{typ}}}{\sigma_s}$$
Inhomogeneity and glassy physics

The emergent mesoscopic inhomogeneity is a signature of “glassy” superconductivity?

Can we make predictions on currents and superfluid density?

NbN films
M. Chand et al
PRB 2012

Increasing disorder

\( T_c \approx 1.65 \text{K} \)

\( k_f \) (at 285K)

Inhomogeneity
Current response in a disordered SC

• It is a tricky job, even within MF: in a clean system \( J_q = \chi_{\text{BCS}}(q) A_q \) and \( \chi_{\text{BCS}}(q \to 0) \) gives the superfluid density \( D_S \) with \( \chi_{\text{BCS}} \) given by the bubble expression with no vertex corrections.

• \( \chi_{\text{BCS}} \) is not gauge invariant, but it is enough since a transverse \( A \) does not change the phase of the SC order parameter \( \Delta \)

• In the presence of disorder this is wrong! We solve the BdG eqs in the presence of \( A \) and evaluate the related local current density \( J(r) \)
Current patterns

\[ H = -t \sum_{\langle ij \rangle, \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + \sum_{i, \sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \]

\[ \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle = |\Delta_i| e^{i\theta_i} \]

- Current in the presence of a finite transverse \( A \), by allowing for the local phases \( \theta_i \) of the BdG solutions to relax to the applied field \( A \).

\( |U| = 5t, V_0 = 2t, n = 0.85 \)
\( \text{size=20x20} \)
\( \text{map: local } \Delta \)
\( \text{lines: constant phase} \)
\( \nabla \theta_i - 2A \)
\( \text{arrows: local current} \)

Unidimensional patterns for the current: glassy-like behavior
Current patterns

\[ H = -t \sum_{<ij>,\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \]

\[ \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle = |\Delta_i| e^{i\theta_i}. \]

- Current in the presence of a finite transverse field \( A \), by allowing for the local phases \( \theta_i \) of the BdG solutions to relax to the applied field \( A \).

\[ D_s^{BCS} \gg D_s = 1/L^2 \partial^2 E(A)/\partial A^2 \]

Missing superfluid density \( D_s^{BCS} - D_s \) is the spectral weight of intragap \( \sigma(\omega) \) from collective modes.

Unidimensional patterns for the current: glassy-like behavior

G. Seibold, L. Benfatto, J. Lorenzana and C. Castellani
PRL 108, 207004 (2012)
Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: *in-gap* spectral weight due to phase fluctuations

Increasing disorder

Increasing SC coupling

$K_{\text{full}} = \text{J} = \text{J}$

$\text{J}:$ collective modes (phase, amplitude, charge)
Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: \textit{in-gap} spectral weight due to phase fluctuations

$\text{Increasing disorder}$

$\text{Increasing SC coupling}$

\textbf{Standard Mattis-Bardeen}

\textbf{Spectral gap $\Delta$, i.e. the same found in the DOS and measured by STM}
Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: \textit{in-gap} spectral weight due to phase fluctuations

Collective-modes contribution at low energies

“Smaller” optical gap than STM gap? Anomalies in G-THz Spectroscopy?
Conclusions

• Disordered superconductors (near SIT): glassy physics?
• Anomalous P(Δ) distribution
• Quasi-1D current paths
• Physical view: coupled SC islands with large variation of Josephson couplings, competition between localization of Cooper pair and coherence
• Open problems: dynamics (optics), insulating phase and critical behavior at SIT
Thickness tuned SIT: amorphous Bi films

Jim Valles, Leiden 2011
Thickness tuned SIT: amorphous Bi films

3 Flavors of SIT

<table>
<thead>
<tr>
<th>Flavor</th>
<th>SIT</th>
<th>Insulator</th>
<th>Transport</th>
<th>MR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular</td>
<td>Bosonic</td>
<td>Localized CP’s</td>
<td>Quasi-particle tunneling</td>
<td>Giant negative</td>
</tr>
<tr>
<td>Uniform</td>
<td>Fermionic</td>
<td>Weakly localized e−’s</td>
<td>Weak localization/ ee interaction type</td>
<td>Small positive</td>
</tr>
<tr>
<td>NHC</td>
<td>Bosonic</td>
<td>Localized CP’s</td>
<td>Incoherent CP tunneling</td>
<td>Giant positive</td>
</tr>
</tbody>
</table>
Thickness tuned SIT: amorphous Bi films

Nano-honeycomb Films

Jim Valles
Lorentz Center, Leiden 2011
Thickness tuned SIT: amorphous Bi films

\[ R = R_0 e^{T_0/T} \]

\textbf{SIT: Activation Energy Goes to Zero}

Data from 4 series’ of films

Activated behaviour in amorphous indium oxyde films

Gantmakher et al. JETP82, 951 (1996)

Shahar and Ovadyahu PRB46, 10917 (1992)

$T_0 \xrightarrow{\text{? gaped insulator}} T_c \xrightarrow{\text{? superconductivity related}}$
Huge magnetoresistance peak: superconductivity-related?

Superconductor-Insulator Transition

Titanium nitride thin films


Activated behaviour in insulating films: superconductivity-related?
Superconductor-Insulator Transition

Activated regime:

\[ \Delta = 1.76 T_c \]

gap in the density of states?

Non-monotonous magneto-resistance in insulators:

superconducting correlations?

Current response in a disordered SC

$|U| = 5t, \ V_0 = 2t, \ n = 0.85$

size = 20x20

small $A_x$ (linear regime)

color map for $|\Delta_i|$

lines = constant phase

arrows = strength of current on links

Almost 1d path
Current response in a disordered SC

Current from BCS response

It does not obey continuity equation

\[ D_{s_{\text{BCS}}} \gg D_s = \frac{1}{L^2} \frac{\partial^2 E(A)}{\partial A^2} \]

Missing superfluid density

\[ D_{s_{\text{BCS}}} - D_s = \text{spectral weight of intragap } \sigma(\omega) \text{ from collective modes} \]
Current response in a disordered SC

Superfluid density from

\[ D_s = \frac{1}{L^2} \frac{\partial^2 E(A)}{\partial A^2} \]

\[ |U| = 5t, \; n=0.85 \text{ vs } V_0 \]

Average over 20 ÷ 40 disorder configurations

\[ \frac{Q}{D_s} \text{ measure of anharmonic terms} \]
Current response in a disordered SC

Distributions of the superfluid density with increasing lattice size

$|U| = 5t$, $V_0 = 2t$

$n = 0.85$
Current response in a disordered SC

$|U| = 5t$, $V_0 = 2t$, $n = 0.1$

size = 20 x 20

small $A_x$ (linear regime)

color map for $|\Delta_i|$

arrows = strength of current on links

Almost 1d path
Current response in a disordered SC

$|U| = 5t$, $V_0 = 2t$, $n = 0.1$

size = 20x20

small $A_x$ (linear regime)

color map for charge $n_i$

arrows = strength of current on links

Almost 1d path
Current response in a disordered SC

Gap - charge correlation
Current response in a disordered SC
Current response in a disordered SC
Disorder driven Superconductor-Insulator transition in 3D NbN
Mondal et. al (2011); Chand et al. (2012)
Define peak height $h=(G_{\text{peak}}-G_{\text{min}})/G_{\text{min}} \propto \text{SC order parameter } s$